1. Zadatak

Za date konačne jednačine kretanja u Dekartovim koordinatama tačke M u metrima i vremena u sekundama odrediti: brzinu i ubrzanje tačke M u proizvoljnom vremenu, a potom tangencijalno i normalno ubrzanje tačke M u trenutkiu t=π/6.

$$x(t) := 2\cdot\cos(2t)$$

$$y(t) := 8\cdot\sin(t)$$

Putanja tačke M:

$$s(t) := \sqrt{(x(t))^2 + (y(t))^2} \qquad s(t) \rightarrow 2\cdot\sqrt{\cos(2\cdot t)^2 + 16\cdot\sin(t)^2}$$

za trenutak vremena: $t_1 := \frac{\pi}{6}$ od $T := 2\pi$

$$x(t_1) = 1 \text{ m} \qquad y(t_1) = 4 \text{ m}$$

$$s(t_1) = 4.123 \text{ m}$$

Brzina tačke M:

$$v_x(t) := \frac{d}{dt}x(t) \qquad v_x(t) \rightarrow -4\cdot\sin(2\cdot t)$$

$$v_y(t) := \frac{d}{dt}y(t) \qquad v_y(t) \rightarrow 8\cdot\cos(t)$$

$$v(t) := \sqrt{(v_x(t))^2 + (v_y(t))^2} \qquad v(t) \rightarrow 4\cdot\sqrt{\sin(2\cdot t)^2 + 4\cdot\cos(t)^2}$$

Ubrzanje tačke M:

$$a_x(t) := \frac{d}{dt}v_x(t) \qquad a_x(t) \rightarrow -8\cdot\cos(2\cdot t)$$

$$a_y(t) := \frac{d}{dt}v_y(t) \qquad a_y(t) \rightarrow -8\cdot\sin(t)$$

$$a(t) := \sqrt{(a_x(t))^2 + (a_y(t))^2} \qquad a(t) \rightarrow 8\cdot\sqrt{\cos(2\cdot t)^2 + \sin(t)^2}$$

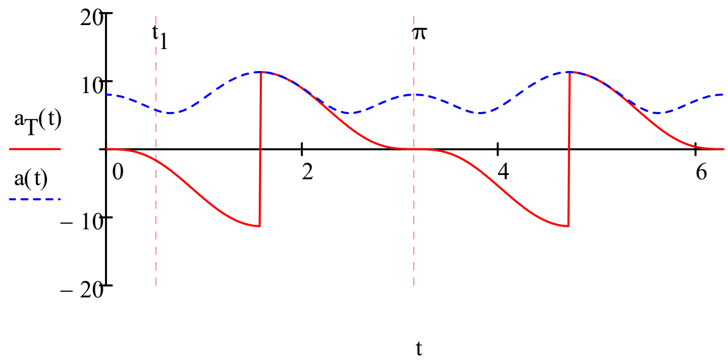
Ukupnog ubrzanja tačke M je: $a(t) = \sqrt{(a_T(t))^2 + (a_N(t))^2}$

odavde najpre odredimo komponentu **tangencijalnog ubrzanja**:

$$a_T(t) := \frac{d}{dt}v(t)$$

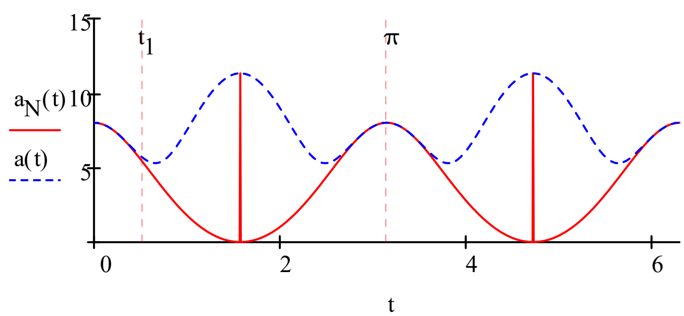
$$a_T(t) \rightarrow \frac{8\cdot\cos(2\cdot t)\cdot\sin(2\cdot t) - 16\cdot\cos(t)\cdot\sin(t)}{\sqrt{\sin(2\cdot t)^2 + 4\cdot\cos(t)^2}}$$

$$a_T(t_1) \text{ simplify } \rightarrow -\frac{4\cdot\sqrt{5}}{5} \qquad a_T(t_1) = -1.789$$

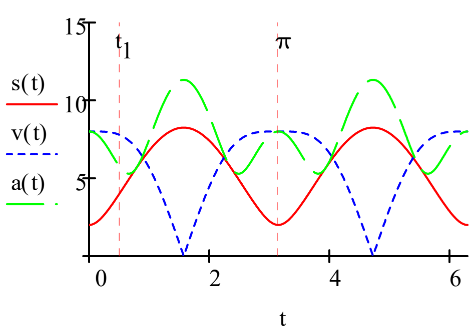


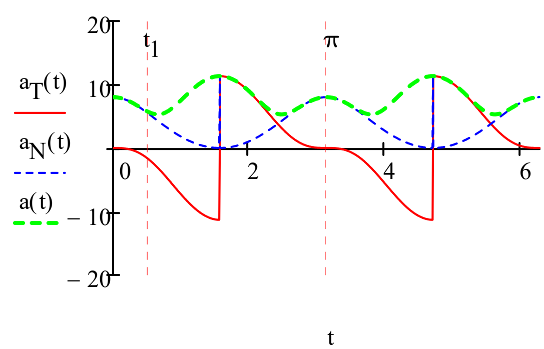
na kraju nalazimo komponentu **normalnog ubrzanja** da je:

$$a_N(t) := \sqrt{(a(t))^2 - (a_T(t))^2} \qquad a_N(t_1) \rightarrow \frac{12 \cdot \sqrt{5}}{5} \qquad a_N(t_1) = 5.367$$



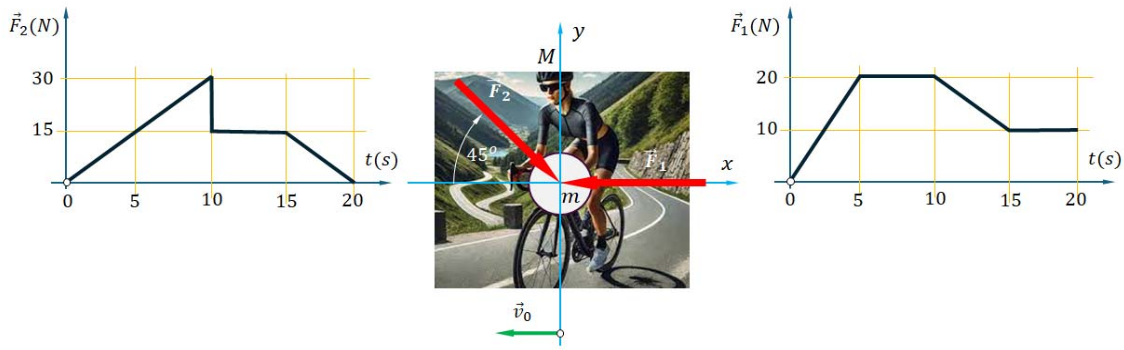
Rekapitulacija





2. Zadatak

Na osnovu date skice, primenom zakona o promeni količine kretanja sa zanemarivanjem težine materijalne tačke usled dejstva sila F1 i F2, sračunati: pravac vektora brzine tačke M, kao i njegov intenzitet u trenutku t=20s.



Sračunavanje pravaca i smerova impulsa sila jer su oni kolinearni sa silama:

$$I_1 = \int_{t_0}^{t_1} F_1\, dt \qquad I_1 := \frac{20\cdot 5}{2} + 20\cdot 5 + \frac{20+10}{2}\cdot 5 + 10\cdot 5 \qquad I_1 = 275$$

$$I_2 = \int_{t_0}^{t_1} F_2\, dt \qquad I_2 := \frac{30\cdot 10}{2} + 15\cdot 5 + \frac{15\cdot 5}{2} \qquad I_2 = 262.5$$

Primenom zakona o promeni količine kretanja nalazimo projekcije sila:

$$\text{Given} \qquad m := 100 \qquad \alpha := 0 \qquad v_{ox} := 10 \qquad \beta := 45 \qquad v_{oy} := 0$$

x-pravac: $m\cdot v_{1x} - m\cdot v_{ox} = -I_1\cdot \cos(\alpha\cdot deg) + I_2\cdot \cos(\beta\cdot deg)$

y-pravac: $m\cdot v_{1y} - m\cdot v_{oy} = I_1\cdot \sin(\alpha\cdot deg) - I_2\cdot \sin(\beta\cdot deg)$

Rešenja sistema jednačina:

$$\text{Find}(v_{1x}, v_{1y}) \rightarrow \begin{pmatrix} \frac{21\cdot\cos(45\cdot deg)}{8} + \frac{29}{4} \\ -\frac{21\cdot\sin(45\cdot deg)}{8} \end{pmatrix}$$

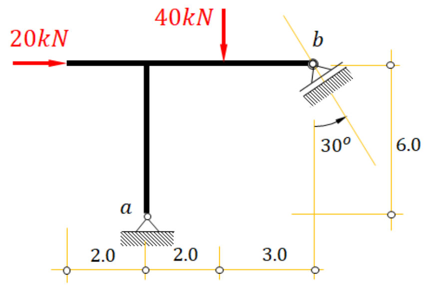
$$v_{1x} := \frac{21\cdot\cos(45\cdot deg)}{8} + \frac{29}{4} \qquad v_{1x} = 9.106$$

$$v_{1y} := -\frac{21\cdot\sin(45\cdot deg)}{8} \qquad v_{1y} = -1.856$$

Intenzitet brzine: $v_M := \sqrt{(v_{1x})^2 + (v_{1y})^2} \qquad v_M = 9.293\ \frac{m}{s}$

3. zadatak

Za nosač na skici primenom opšte jednačine statike sračunati rekacije veza u čvoru "a".



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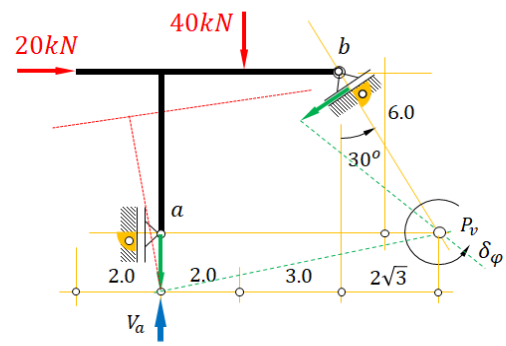
1. plan pomeranja:



$$\delta\phi := \frac{1}{\left(2 + 3 + 2\sqrt{3}\right)}$$

$$\delta\phi = 0.118$$

$$\delta A = 0$$

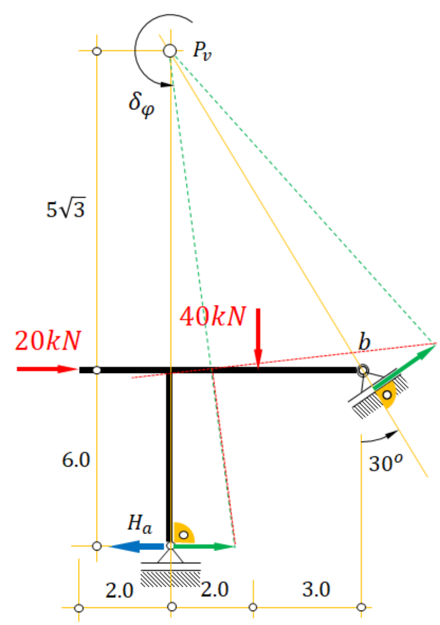
Given

$$-Va\cdot\delta\phi\cdot\left(2 + 3 + 2\cdot\sqrt{3}\right) - 20\cdot\delta\phi\cdot 6 + 40\cdot\delta\phi\cdot\left(3 + 2\sqrt{3}\right) = 0$$

$$\text{Find}(Va) \rightarrow \frac{80\cdot\sqrt{3}}{2\cdot\sqrt{3} + 5} \qquad Va := \frac{80\cdot\sqrt{3}}{2\cdot\sqrt{3} + 5}$$

$$Va = 16.371$$

2. plan pomeranja:



$$\delta\phi := \frac{1}{\left(6 + 5\sqrt{3}\right)}$$

$$\delta\phi = 0.068$$

$$\delta A = 0$$

Given

$$-Ha\cdot\delta\phi\cdot\left(6 + 5\cdot\sqrt{3}\right) + 20\cdot\delta\phi\cdot 5\sqrt{3} - 40\cdot\delta\phi\cdot 2 = 0$$

$$\text{Find}(Ha) \rightarrow \frac{20\cdot\left(5\cdot\sqrt{3} - 4\right)}{5\cdot\sqrt{3} + 6} \qquad Ha := \frac{20\cdot\left(5\cdot\sqrt{3} - 4\right)}{5\cdot\sqrt{3} + 6}$$

$$Ha = 6.358$$